SOLUZIONI

$\boxed{x^2=y}$

$\boxed{3}$ $\int_0^{+\infty} \frac{3x}{x^4+5x^2+4}\,dx = \lim_{b\to+\infty} \frac{3}{2}\int_0^b \frac{1}{x^4+5x^2+4}\underbrace{\boxed{2x}\,dx}_{(x^2)'} = \lim_{b\to+\infty} \frac{1}{2}\int_0^{b^2} \frac{3}{\underbrace{y^2+5y+4}}\,dy =$

$y^2+y+4y+4$

$(y+4)(y+1)$

$\frac{y+4-1-y}{(y+4)(y+1)}$

$\frac{1}{y+1} - \frac{1}{y+4}$

$= \boxed{\lim_{c\to+\infty} \frac{1}{2}\int_0^c \left(\frac{1}{y+1} - \frac{1}{y+4}\right)dy =}$

$= \lim_{c\to+\infty} \frac{1}{2}\left[\ln\frac{y+1}{y+4}\right]_0^c =$

$= \lim_{c\to+\infty} \left(\underbrace{\frac{1}{2}\ln\left(\frac{c+1}{c+4}\right)}_{\to 0} - \frac{1}{2}\ln\left(\frac{1}{4}\right)\right) = +\frac{1}{2}\ln 4 = \boxed{\ln 2}$

NO

$= \lim_{c\to+\infty}\left(\frac{1}{2}\int_0^c \frac{1}{y+1}\,dy - \frac{1}{2}\int_0^c \frac{1}{y+4}\,dy\right) = +\infty - \infty = (???)$

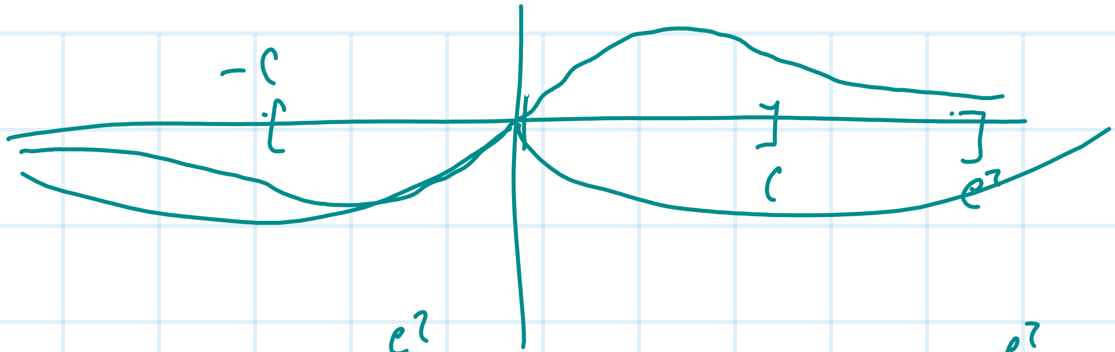
$\boxed{4}$ $\frac{1}{2}\int_{-\infty}^{+\infty} \frac{2x}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{1}{2}\frac{2x}{1+x^2}\,dx + \int_0^{+\infty} \frac{1}{2}\frac{2x}{1+x^2}\,dx = \lim_{c\to-\infty}\left[\ln\sqrt{1+x^2}\right]_c^0 + \lim_{c\to+\infty}\left[\ln\sqrt{1+x^2}\right]_0^c =$

$\ln\sqrt{1+x^2}$

$= -\infty + \infty$ NON CONVERGE

NO

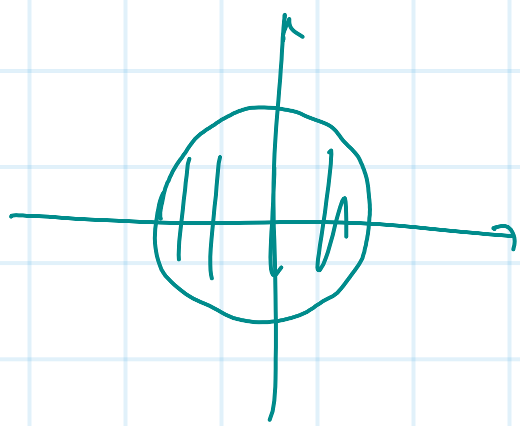
$\lim_{c\to+\infty}\int_{-c}^{c} \frac{2x}{1+x^2}\,dx = \lim_{c\to+\infty} 0 = 0$ NO

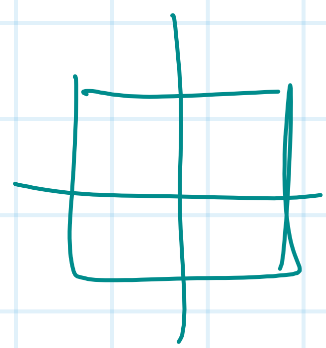


$$\lim_{c\to+\infty} \int_{-c}^{e^?} \frac{1}{2}\frac{2x}{x^2+1}\,dx = \lim_{c\to+\infty} \left[\ln\sqrt{1+x}\right]_c^{e^?} =$$

$$= \lim_{c\to+\infty}\left(\ln\sqrt{1+e^?} - \ln\sqrt{1+c}\right) =$$

$$\lim_{c\to+\infty} \ln\sqrt{\frac{1+e^?}{1+c}} = -\infty$$





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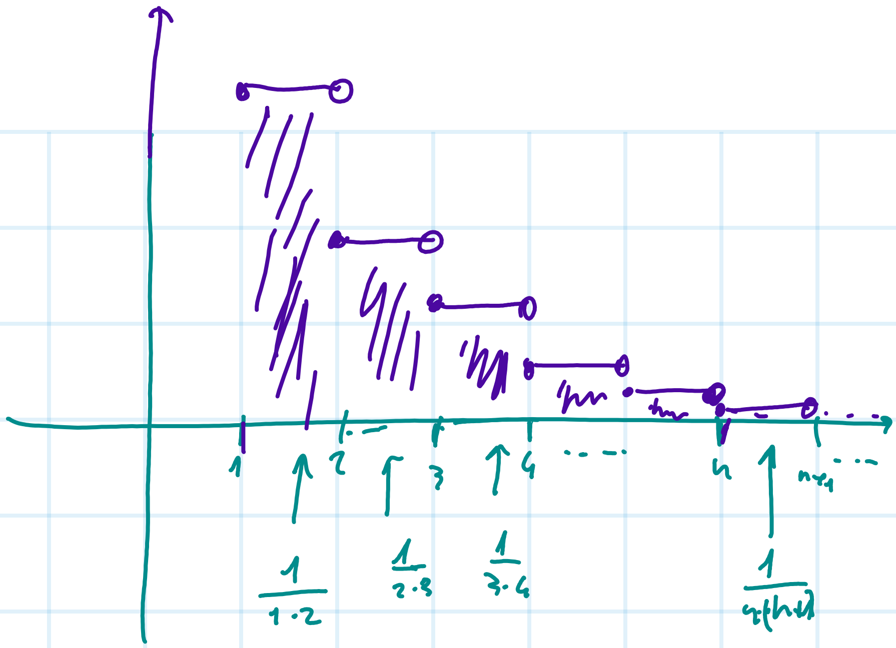
9 $\int_1^{+\infty} \frac{1}{\lfloor x\rfloor\cdot\lfloor x+1\rfloor}\,dx =$ $\qquad F(x) = \int_1^x \underbrace{f(t)}_{>0}\,dt$ $\qquad F(x+\delta) - F(x) = \int_x^{x+\delta} f(t)\,dt$

$\lim_{x\to+\infty} F(x) =$ $\qquad \lim_{b\to+\infty}\int_1^b \frac{1}{\lfloor x\rfloor\cdot\lfloor x+1\rfloor}\,dx =$

$a_n \to +\infty$

$\lim_{n\to+\infty} F(a_n) =$ $\qquad = \lim_{n\to+\infty}\int_1^{n+1}\frac{1}{\lfloor x\rfloor\cdot\lfloor x+1\rfloor}\,dx =$



$$\lim_{n\to+\infty}\int_1^{n+1} f(t)\,dt = \int_1^2 + \int_2^3 + \dots + \int_n^{n+1} f(t)\,dt =$$

$$= \lim_{n\to+\infty} \underbrace{\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{n(n+1)}}$$

$$= \frac{2-1}{1\cdot 2} + \frac{3-2}{2\cdot 3} + \frac{4-3}{3\cdot 4} + \dots \frac{(n+1)-n}{n(n+1)} =$$

$$\frac{2}{1\cdot 2} - \cancel{\frac{1}{1\cdot 2}} + \cancel{\frac{3}{2\cdot 3}} - \cancel{\frac{2}{2\cdot 3}} + \cancel{\frac{4}{3\cdot 4}} - \frac{3}{3\cdot 4} + \dots + \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)}$$

$$\frac{1}{1} - \frac{1}{n+1} = \frac{n}{n+1} \xrightarrow{n\to+\infty} 1$$

$\boxed{11}$ $\displaystyle\int_0^{+\infty} \frac{\sin^4 x + \cos^4 x}{\ln(1+e^x)}\,dx$

$\displaystyle\int \frac{1}{\ln(1+e^x)}$ $\quad \frac{1}{x}$

$\displaystyle\int_1^{+\infty} \frac{1}{x}\,dx = +\infty$

$$\forall x \in \mathbb{R} \quad f(x) \geq \boxed{\frac{m}{\ln(1+e^x)}}$$

$m = \min\{\sin^4 x + \cos^4 x \mid x \in [0, +\infty[\}$

$$\ln(1+e^x) = \ln\left(e^x\left(\frac{1}{e^x}+1\right)\right) = \ln(e^x) + \ln\left(1+\frac{1}{e^x}\right) =$$

$$= x + \underbrace{\ln\left(1+\frac{1}{e^x}\right)}$$

$$\lim_{x\to+\infty} \frac{x}{\ln(1+e^x)} = \lim_{x\to+\infty} \frac{x}{x+\underbrace{\ln\left(1+\frac{1}{e^x}\right)}_{0}} = \lim_{x\to+\infty} \frac{1}{1+\underbrace{\frac{1}{x}\ln\left(1+\frac{1}{e^x}\right)}_{0}} = 1$$

$$\boxed{\int_1^{+\infty} \frac{m}{\ln(1+e^x)}\,dx} \quad \text{DIVERGE}$$

$$\boxed{\frac{m}{\ln(1+e^x)}} \leq \boxed{\frac{\cos^4 x+\sin^4 x}{\ln(1+e^x)}}$$

$$\boxed{\int_1^{+\infty} \frac{m}{x}\,dx} \quad \text{DIVERGE}$$

$\boxed{11}$ DIVERGE

$\boxed{12}$ $$\int_0^{+\infty} \frac{\sin(x^4)+\cos(x^4)}{\ln(1+e^x)}\,dx = \underbrace{\int_0^{+\infty} \frac{\sin(x^4)}{\ln(1+e^x)}\,dx} + \int_0^{+\infty} \frac{\cos(x^4)}{\ln(1+e^x)}\,dx$$

$$\int \frac{\sin x}{x}\,dx$$

HA PRIMITIVA LIMITATA

TENDE A 0 DECRESCENDO

$$\int_0^{+\infty} \frac{\boxed{\sin(x^4)}}{\ln(1+e^x)}\,dx = \int_0^{+\infty} \boxed{\sin(x^4)} \cdot \boxed{\frac{1}{\ln(1+e^x)}}\,dx$$

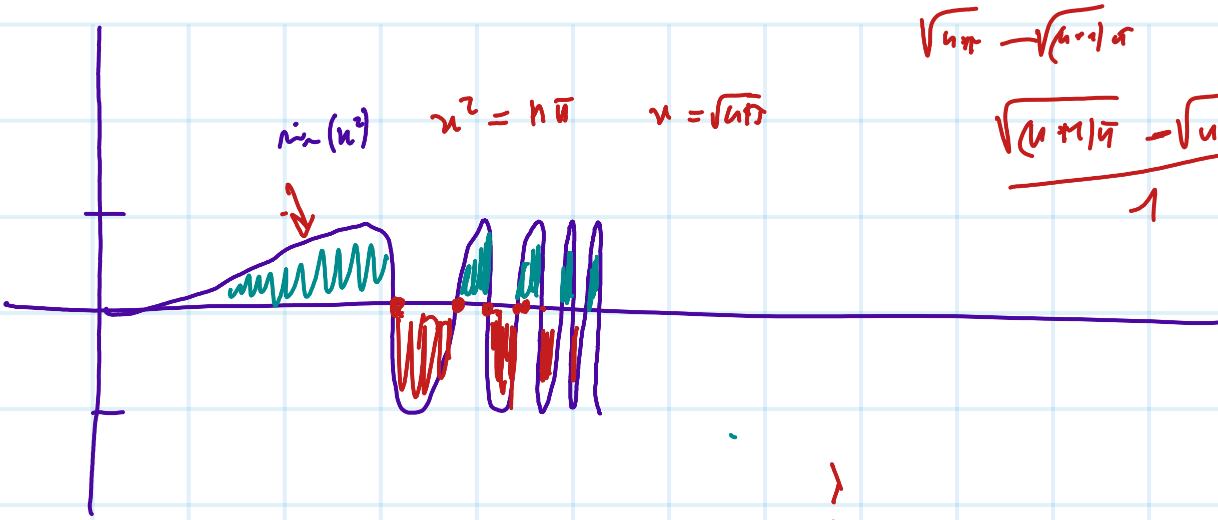
$$\lim_{b\to+\infty} \int_0^b \sin(x^4)\,dx = \int_0^{+\infty} \sin(x^4)\,dx < +\infty$$

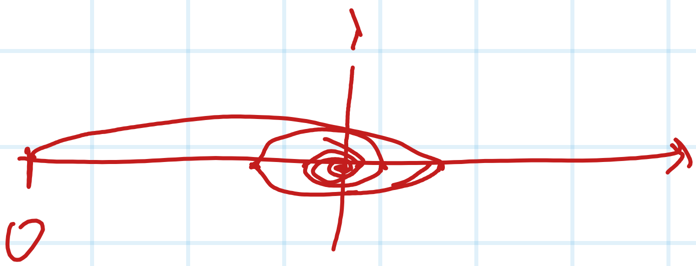
$$F(x) = \int_0^x \sin(t^4)\,dt$$

$\sqrt{n\pi} - \sqrt{(n+1)\pi}$

$\sqrt{(n+1)\pi} - \sqrt{n\pi} = \dfrac{\pi}{\sqrt{(n+1)\pi} + \sqrt{n\pi}}$

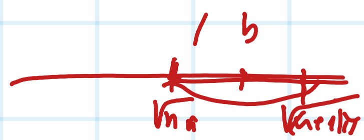
$\rightarrow 1$





$F(b) - F(\sqrt{n\pi}) \to 0$

$$\lim_{b \to +\infty} \int_0^b \sin(x^2)\,dx$$



$$\lim_{n \to +\infty} \int_0^{\sqrt{n\pi}} \sin(x^2)\,dx$$

$x = \sqrt{t}$

$$\lim_{b \to +\infty} \int_0^b \sin(x^2)\,dx = \lim_{b \to +\infty} \int_0^{b^2} \sin\left((\sqrt{t})^2\right) \cdot \frac{1}{2\sqrt{t}}\,dt =$$

$$\int_0^{+\infty} \sin(x^2)\,dx < +\infty$$

$$= \lim_{c \to +\infty} \int_0^c \frac{\sin t}{2\sqrt{t}}\,dt =$$

$$\int_0^{+\infty} \frac{\sin t}{2\sqrt{t}}\,dt \text{ converge?}$$

$$\boxed{\int_0^{+\infty} \sin(x^4)\,dx} = \lim_{b\to+\infty}\int_0^b \sin(x^4)\,dx \overset{x=\sqrt[4]{t}}{=} \lim_{b\to+\infty}\int_0^{b^4}\sin t\cdot\frac{1}{4\sqrt[4]{t^3}}\,dt =$$

$$= \int_0^{+\infty}\frac{\sin t}{4\sqrt[4]{t^3}}\,dt$$ CONVERGE

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15 $\int_0^{+\infty}\frac{\arctan(\sin x)}{\ln(e+x+\sin x)}\,dx$

$\int_0^{2\pi}\arctan(\sin x)\,dx \overset{?}{=} 0$

$\int_{-\pi}^{\pi}$

$\frac{1}{\ln(e+x+\sin x)} \searrow 0$

$h(x)$ periodica T media nulla $\int_0^T h(x)\,dx = 0$

$F(x)=\int_0^x h(t)\,dt$

$\boxed{F(x+T)\overset{?}{=}F(x)}$ $\forall x\in\mathbb{R}$

$\int_0^{x+T} h(t)\,dt \overset{?}{=} \int_0^x h(t)\,dt$

$\int_0^{x+T} - \int_0^x = \int_x^{x+T} dt = 0$

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16 $\int_0^{+\infty} \boxed{\frac{1}{x}}\,\boxed{\ln\left(1+\frac{1}{2}\sin x\right)}\,dx$

$\int_0^{2\pi}\ln\left(1+\frac{1}{2}\sin x\right)dx < 0$

$f(y) = \ln(1+y)$

$g(x) = \frac{1}{2}\sin x$

$f(g(x)) = \ln\left(1+\frac{1}{2}\sin x\right)$

$\int f(g(x))\,dx$